L-functions and applications
1. Exercise Sheet

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Groupwork

Exercise G1
Show that there are infinitely many prime numbers congruent to 3 modulo 4 and infinitely many prime numbers congruent to 1 modulo 4.

Hint: Try to imitate Euclid's proof of the infinitude of prime numbers.

Exercise G2
Use the Euler product of the zeta function to show

$$\sum_{p\in\mathbb{P},p<N} \frac{1}{p} = \log\log N + O(1)$$

as $N \to \infty$. **Hint:** The following fact about the harmonic series can be proved by approximating the sum by integrals:

$$\sum_{n\in\mathbb{N}} \frac{1}{n} = \log N + O(1)$$

as $N \to \infty$.

Exercise G3
Let p_n be the n-th prime number. Show that for any $k > 1$ and any N, there exists $n > N$ with $p_n < kn\log n$.